AP Calculus BC — Polar Equations & Motion

1) $r = 3 + 3\sin(2\theta)$

$x = (3 + 3\sin(2\theta))\cdot\cos\theta$

$\frac{dx}{d\theta} = -(3 + 3\sin(2\theta))\sin\theta + (6\cos(2\theta))\cos\theta$

$\left.\frac{dx}{d\theta}\right|_{\theta=\frac{\pi}{3}} = -\left(3 + 3\left(\frac{\sqrt{3}}{2}\right)\right)\frac{\sqrt{3}}{2} + (-3)\left(\frac{1}{2}\right)$

2) $r(\theta) = 3\cos\theta \quad r'(\theta) = -3\sin\theta$

$x = 3\cos^2\theta \quad y = 3\cos\theta\sin\theta$

$\frac{dx}{d\theta} = -6\cos\theta\sin\theta \quad \frac{dy}{d\theta} = 3\cos^2\theta - 3\sin^2\theta$

$\frac{dy}{dx} = \frac{3\cos^2\theta - 3\sin^2\theta}{-6\cos\theta\sin\theta}$

$\left.\frac{dy}{dx}\right|_{\theta=\frac{\pi}{4}} = 0$

3) $r = -1 + \sin\theta \quad r' = \cos\theta$

$x = (-1+\sin\theta)\cos\theta \quad y = (-1+\sin\theta)\sin\theta$

$x(\pi) = 1 \quad y(\pi) = 0 \quad (1, 0)$

$\frac{dx}{d\theta} = -(-1+\sin\theta)\sin\theta + \cos^2\theta \quad \frac{dy}{d\theta} = (-1+\sin\theta)\cos\theta + \cos\theta\sin\theta$

$\frac{dy}{dx} = \frac{-\cos\theta + 2\cos\theta\sin\theta}{\sin\theta - \sin^2\theta + \cos^2\theta} \quad \left.\frac{dy}{dx}\right|_{\theta=\pi} = \frac{-1}{1} = -1$

$\boxed{y - 0 = -1(x-1)}$

4) $r = 4 - \sin(3\theta)$

$\frac{dr}{dt} = -3\cos(3\theta)\frac{d\theta}{dt}$

$\left.\frac{dr}{dt}\right|_{\theta=\frac{\pi}{6}} = -3\left(\frac{\pi}{2}\right)(3) = 0$

5) a) $r = 4 - 2\sin\theta$

$x = (4-2\sin\theta)\cos\theta \quad y = (4-2\sin\theta)\sin\theta$

$s(t) = \langle (4-2\sin t^2)\cos t^2, (4-2\sin t^2)\sin t^2 \rangle$

$v(1.5) = \langle -8.072, -1.6729 \rangle$

b) $(4-2\sin t^2)\cos t^2 = -1$

$t = 1.4279$

6) $\frac{dx}{d\theta} = \cos\theta - \theta\sin\theta \qquad \frac{dy}{d\theta} = \sin\theta + \theta\cos\theta$

$$\frac{dy}{dx} = \frac{\sin\theta + \theta\cos\theta}{\cos\theta - \theta\sin\theta}$$

$$\frac{d^2y}{dx^2} = \frac{(\cos\theta - \theta\sin\theta)(\cos\theta - \theta\sin\theta + \cos\theta) - (\sin\theta + \theta\cos\theta)(-\sin\theta - \theta\cos\theta - \sin\theta)}{(\cos\theta - \theta\sin\theta)^3}$$

$$\frac{d^2y}{dx^2} = \frac{\left(\frac{3\pi}{2}\right)\left(\frac{3\pi}{2}\right) - (-1)(2)}{\left(\frac{3\pi}{2}\right)^3} = \frac{\frac{9\pi^2}{4} + 2}{\frac{27\pi^3}{8}}$$

$$= \frac{\frac{9\pi^2}{4} + \frac{8}{4}}{\frac{27\pi^3}{8}} = \frac{18\pi^2 + 16}{27\pi^3}$$